MECHANICAL DRAWING
PART II

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INSTRUCTION PAPER

PREPARED BY

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MECHANICAL DRAWING

PART II

GEOMETRICAL DEFINITIONS

A *point* is used for marking position; it has neither length, breadth, nor thickness.

LINES

A *line* has length only; it is produced by the motion of a point.

A *straight line* or *right line* is one that has the same direction throughout. It is the shortest distance between two points.

A *curved line* is one that is constantly changing in direction. It is sometimes called a curve.

A *broken line* is one made up of several straight lines.

Parallel lines are lines which lie in the same plane and are equally distant from each other at all points.

A *horizontal line* is one having the direction of a line drawn upon the surface of water that is at rest. It is a line parallel to the horizon.

A *vertical line* is one that lies in the direction of a thread suspended from its upper end and having a weight at the lower end. It is a line that is perpendicular to a horizontal plane.

An *oblique line* is one that is neither vertical nor horizontal.

In Mechanical Drawing, lines drawn along the edge of the T-square, when the head of the T-square is resting against the left-hand edge of the board, are called *horizontal* lines. Those drawn at right angles or perpendicular to the edge of the T-square are called *vertical* lines.

If two lines cut each other, they are called *intersecting lines*, and the point at which they cross is called the *point of intersection*.

ANGLES

An *angle* is the measure of the difference in direction of two lines. The lines are called *sides*, and the point of meeting, the *vertex*. The size of an angle is independent of the length of the lines.

If one straight line meets another (extended if necessary), Fig. 40, so that the two angles thus formed are equal, the lines are said
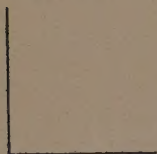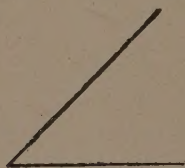

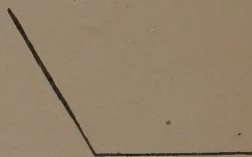Fig. 40.   Right Angle   Fig. 41.   Acute Angle   Fig. 42.   Obtuse Angle

to be *perpendicular* to each other and the angles formed are called *right angles*.

An *acute angle* is less than a right angle, Fig. 41.

An *obtuse angle* is greater than a right angle, Fig. 42.

SURFACES

A *surface* is produced by the motion of a line; it has two dimensions—length and breadth.

A *plane figure* is a plane bounded on all sides by lines; the space included within these lines (if they are straight lines) is called a *polygon* or a *rectilinear figure*.

POLYGONS

A *polygon* is a plane figure bounded by straight lines. The
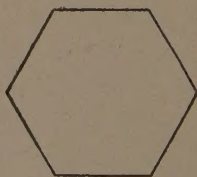

Fig. 43.   Pentagon   Fig. 44.   Hexagon   Fig. 45.   Octagon

boundary lines are called the *sides* and the sum of the sides is called the *perimeter*.

Polygons are classified according to the number of sides.

A *triangle* is a polygon of *three* sides.

A *quadrilateral* is a polygon of *four* sides.

A *pentagon* is a polygon of *five* sides, Fig. 43.

A *hexagon* is a polygon of *six* sides, Fig. 44.

A *heptagon* is a polygon of *seven* sides.

An *octagon* is a polygon of *eight* sides, Fig. 45.

A *decagon* is a polygon of *ten* sides.

A *dodecagon* is a polygon of *twelve* sides.

An *equilateral* polygon is one all of whose sides are equal.

An *equiangular* polygon is one all of whose angles are equal.

A *regular* polygon is one all of whose angles and all of whose sides are equal.

Triangles. A triangle is a polygon enclosed by three straight lines called *sides*. The *angles* of a triangle are the angles formed by the sides.

A *right-angled* triangle, often called a *right* triangle, Fig. 46, is one that has a right angle. The longest side (the one opposite
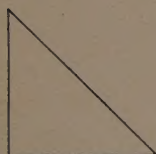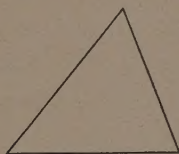
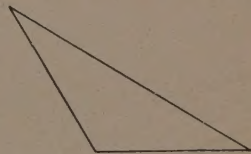Fig. 46.   Right-Angled   Fig. 47.   Acute-Angled   Fig. 48.   Obtuse-Angled
       Triangle   Triangle   Triangle

the right angle) is called the *hypotenuse,* and the other sides are sometimes called *legs.*

An *acute-angled* triangle is one that has all of its angles acute, Fig. 47.

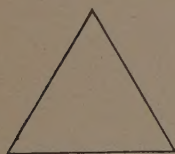An *obtuse-angled* triangle is one that has an obtuse angle, Fig. 48.
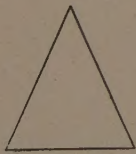
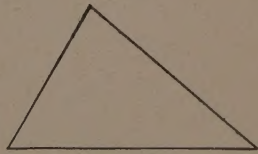Fig. 49. Equilateral Triangle   Fig. 50. Isosceles Triangle   Fig. 51. Scalene Triangle

An *equilateral* triangle is one having all of its sides equal, Fig. 49.

An *equiangular* triangle is one having all of its angles equal.

An *isosceles* triangle, Fig. 50, is one, two of whose sides are equal.

A *scalene* triangle, Fig. 51, is one, no two of whose sides are equal.

The *base* of a triangle is the lowest side; it is the side upon which the triangle is supposed to stand. Any side may, however, be taken as the base. In an isosceles triangle, the side which is not one of the equal sides is usually considered as the base.

The *altitude* of a triangle is the perpendicular drawn from the vertex to the base.

Quadrilaterals. A quadrilateral is a polygon bounded by four straight lines, as Fig. 52.

The *diagonal* of a quadrilateral is a straight line joining two opposite vertices.

Trapezium. A trapezium is a quadrilateral, no two of whose sides are parallel.

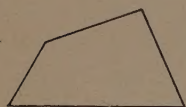*Trapezoid.* A trapezoid is a quadrilateral having two sides
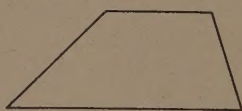
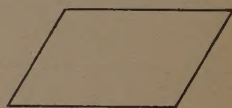Fig. 52. Quadrilateral   Fig. 53. Trapezoid   Fig. 54. Parallelogram

parallel, Fig. 53. The parallel sides are called the *bases* and the perpendicular distance between the bases is called the *altitude*.

Parallelogram. A parallelogram is a quadrilateral whose opposite sides are parallel, Fig. 54.

There are four kinds of parallelograms: rectangle, square, rhombus, and rhomboid.

The *rectangle*, Fig. 55, is a parallelogram whose angles are right angles.
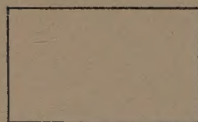
Fig. 55. Rectangle   Fig. 56. Square   Fig. 57. Rhombus

The *square*, Fig. 56, is a parallelogram all of whose sides are equal and whose angles are right angles.

The *rhombus*, Fig. 57, is a parallelogram whose sides are equal but whose angles are not right angles.

The *rhomboid* is a parallelogram whose adjacent sides are unequal, and whose angles are not right angles.

CIRCLES

A *circle* is a plane figure bounded by a curved line called the *circumference*, every point of which is equally distant from a point within called the *center*, Fig. 58.

A *diameter* of a circle is a straight line drawn through the center, terminating at both ends in the circumference, Fig. 59.
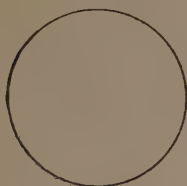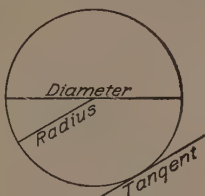

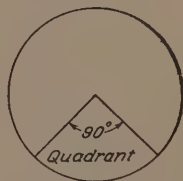Fig. 58.   Circle

Fig. 59.   Diameter and Tangent

Fig. 60.   Quadrant

A *radius* of a circle is a straight line joining the center with the circumference.   All radii of the same circle are equal and their length is always one-half that of the diameter.

An *arc* is any part of the circumference of a circle.   An arc equal to one-half the circumference is called a *semi-circumference*, and an arc equal to one-quarter of the circumference is called a *quadrant*, Fig. 60.   A quadrant may mean the arc or angle.

A *chord*, Fig. 61, is a straight line which joins the extremities of an arc but does not pass through the center of the circle.

A *secant* is a straight line which intersects the circumference in two points, Fig. 61.

A *segment* of a circle, Fig. 62, is the area included between an arc and a chord.

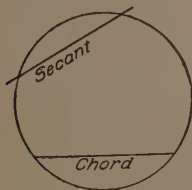A *sector* is the area included between an arc and two radii drawn

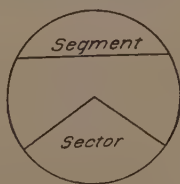Fig. 61. Chord and Secant   Fig. 62   Segment and Sector   Fig. 63. Concentric Circles

to the extremities of the arc, Fig. 62.

A *tangent* is a straight line which touches the circumference at only one point, called the *point of tangency or contact*, Fig. 59.

Concentric circles are circles having the same center, Fig. 63.

An *inscribed angle* is an angle whose vertex lies in the circumference and whose sides are chords. It is measured by one-half the intercepted arc, Fig. 64.
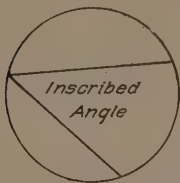

Fig. 64. Inscribed Angle   Fig. 65. Central Angle   Fig. 66. Inscribed Polygon
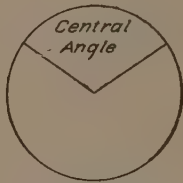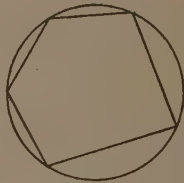
A *central angle* is an angle whose vertex is at the center of the circle and whose sides are radii, Fig. 65.

An *inscribed polygon* is one whose vertices lie in the circumference and whose sides are chords, Fig. 66.

MEASUREMENT OF ANGLES

To measure an angle, take any convenient radius and describe an arc with the center at the vertex of the angle. The portion of the arc included between the sides of the angle is the *measure of the angle*. If the arc has a constant radius, the greater the divergence of the sides, the longer will be the arc. If there are several arcs drawn with the same center, the intercepted arcs will have different lengths but they will all be the *same fraction* of the entire circumference.

In order that the size of an angle or arc may be stated without saying that it is a certain fraction of a circumference, the circumference is divided into 360 equal parts called *degrees*, Fig. 67. Thus, it may be said that a certain angle contains 45 degrees, *i. e.*, it is $\frac{45}{360} = \frac{1}{8}$ of a circumference. In order to obtain accurate measurements each degree is divided into 60 equal parts called *minutes* and each minute into 60 equal parts called *seconds*.

Fig. 67.   Angular Measurement

SOLIDS

A *solid* has three dimensions—length, breadth, and thickness. The most common forms of solids are *polyedrons, cylinders, cones,* and *spheres.*

POLYEDRONS

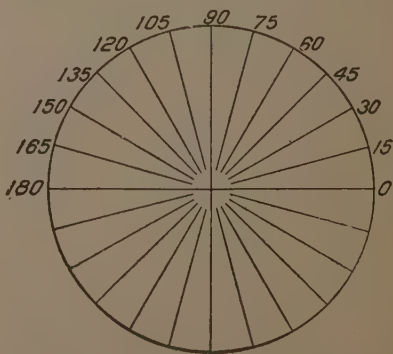A *polyedron* is a solid bounded by planes. The bounding planes are called *faces* and their intersections are called *edges.* The intersections of the edges are called *vertices.*

A polyedron having four faces is called a *tetraedron;* one having six faces, a *hexaedron;* one having eight faces, an *octaedron,* Fig. 68; one having twelve faces, a *dodecaedron,* etc.

Prisms. A prism is a polyedron having two opposite faces, called *bases,* which are equal and parallel, and other faces, called
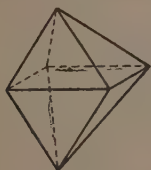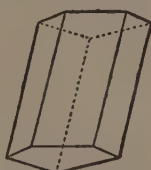
Fig. 68.   Octaedron   Fig. 69.   Prism   Fig. 70.   Right Prism
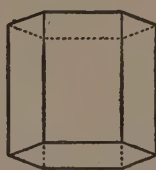
lateral faces, which are parallelograms, Fig. 69. The *altitude* of a prism is the perpendicular distance between the bases. The area of the lateral faces is called the *lateral area.*

Prisms are called *triangular, rectangular, hexagonal,* etc., according to the shape of the bases. Further classifications are as follows:
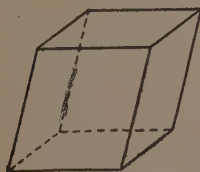
Fig. 71. Parallelopiped   Fig. 72. Rectangular Paral-   Fig. 73. Truncated Prism
                                       lelopiped

A *right* prism is one whose lateral faces are perpendicular to the bases, Fig. 70.

A *regular* prism is a right prism having regular polygons for bases.

Parallelopiped. A parallelopiped is a prism whose bases are

parallelograms, Fig. 71. If all the edges are perpendicular to the bases, it is called a *right parallelopiped.*

A *rectangular parallelopiped* is a right parallelopiped whose bases and lateral faces are rectangles, Fig. 72.

A *cube* is a rectangular parallelopiped all of whose faces are squares.

A *truncated prism* is the portion of a prism included between the base and a plane not parallel to the base, Fig. 73.

Pyramids. A pyramid is a polyedron whose base is a polygon and whose lateral faces are triangles having a common vertex called the *vertex* of the pyramid.

The *altitude* of the pyramid is the perpendicular distance from the vertex to the base.

Pyramids are named according to the kind of polygon forming the base, viz, *triangular, quadrangular,* Fig. 74, *pentagonal,* Fig. 75, *hexagonal.*

A *regular pyramid* is one whose base is a regular polygon and
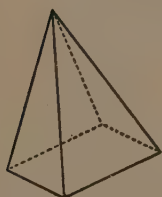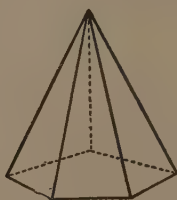
Fig. 74.   Pyramid   Fig. 75.   Regular Pyramid   Fig. 76. Frustum of
                                                                          Pyramid
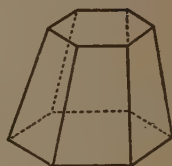
whose vertex lies in a perpendicular erected at the center of the base, Fig. 75.

A *truncated* pyramid is the portion of a pyramid included between the base and a plane not parallel to the base.

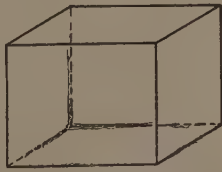A *frustum* of a pyramid is the solid included between the base and a plane parallel to the base, Fig. 76; its *altitude* is the perpendicular distance between the bases.

CYLINDERS

A *cylinder* is a solid having as bases two equal parallel curved surfaces and as its lateral face the continuous surface generated by a straight line connecting the bases and moving along their circumferences. The bases are usually circles and such a cylinder is called a *circular cylinder,* Fig. 77.

A *right cylinder*, Fig. 78, is one whose side is perpendicular to the bases.

The *altitude* of a cylinder is the perpendicular distance between the bases.
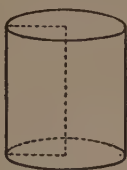
Fig. 77. Cylinder   Fig. 78. Right Cylinder   Fig. 79. Inscribed Cylinder
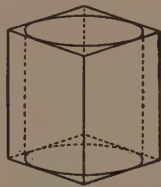
A prism whose base is a regular polygon may be inscribed in or circumscribed about a circular cylinder, Fig. 79.

CONES

A *cone* is a solid bounded by a conical surface and a plane which cuts the conical surface. It may be considered as a pyramid with an infinite number of sides, Fig. 80.

The conical surface is called the *lateral area* and it tapers to a point called the *vertex;* the plane is called the *base.*

The *altitude* of a cone is the perpendicular distance from the vertex to the base.

An *element of the cone* is any straight line from the vertex to the circumference of the base.

A *circular cone* is a cone whose base is a circle

A *right circular cone*, or *cone of revolution*, Fig. 81, is a cone

Fig. 80. Cone   Fig. 81. Right Circular   Fig. 82. Frustum of
                                     Cone   Cone
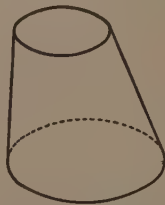
whose axis is perpendicular to the base. It may be generated by the revolution of a right triangle about one of the legs as an axis.

A *frustum* of a cone, Fig. 82, is the portion of the cone included between the base and a plane parallel to the base; its *altitude* is the perpendicular distance between the bases.

SPHERES

A *sphere* is a solid bounded by a curved surface, **every point** of which is equally distant from a point within called the *center*.

The *diameter* is a straight line drawn through the center and having its extremities in the curved surface. The *radius* — $\frac{1}{2}$ diameter —is the straight line from the center to a point on the surface.

A *plane is tangent to a sphere* when it touches the sphere in only
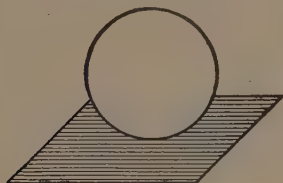
Fig. 83. Plane Tangent to Sphere   Fig. 84. Large Circle   Fig. 85. Small Circle
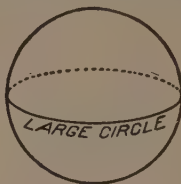
one point. A plane perpendicular to a radius at its outer extremity is tangent to the sphere, Fig. 83.

An *inscribed polyedron* is a polyedron whose vertices lie in the surface of the sphere.

A *circumscribed polyedron* is a polyedron whose faces are tangent to a sphere.

A *great circle* is the intersection of the spherical surface and a plane passing through the center of a sphere, Fig. 84.

A *small circle* is the intersection of the spherical surface and a plane which does not pass through the center, Fig. 85.

CONIC SECTIONS

If a plane intersects a cone the geometrical figures thus formed are called *conic sections*. A plane perpendicular to the base and
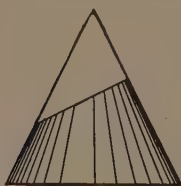
Fig. 86a   Fig. 86b   Fig. 86c   Fig. 86d
Illustrations of Method of Forming Conic Sections.

passing through the vertex of a right circular cone forms an isosceles triangle. If the plane is parallel to the base the intersection of the

plane and conical surfaces will be the circumference of a circle.

Ellipse. An ellipse is a curve formed by the intersection of a plane and a cone, Fig. 86a, or cylinder, Fig. 86b, the plane being oblique to the axis but not cutting the base. An ellipse may be defined as *a curve generated by a point moving in a plane in such a manner that the sum of the distances from the point to two fixed points shall always be constant.*

The two fixed points are called *foci*, Fig. 87, and shall lie on the longest line that can be drawn in the ellipse which is called the
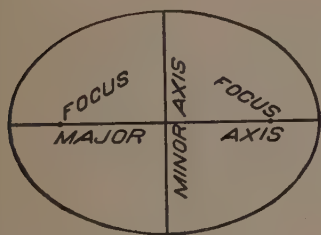
Fig. 87.   Ellipse
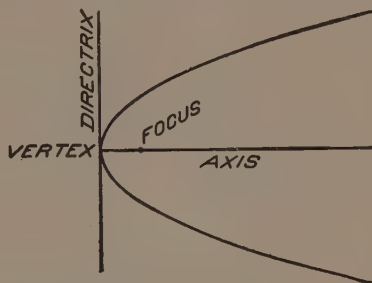
Fig. 88.   Parabola

major axis; the shortest line is called the *minor axis*, and is perpendicular to the major axis at its middle point, called the *center.*

An ellipse may be constructed if the major and minor axes are given or if the foci and one axis are known.

Parabola. The parabola is a curve formed by the intersection of a cone and a plane parallel to an element of the cone, Fig. 86c. This curve is not a closed curve for the branches approach parallelism.

A parabola may be defined as *a curve every point of which is equally distant from a line and a point.*

The point is called the *focus*, Fig. 88, and the given line, the *directrix.* The line perpendicular to the directrix and passing through the focus is the *axis.* The intersection of the axis and the curve is the *vertex.*
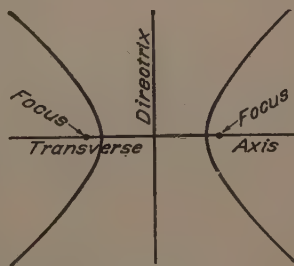
Fig. 89.   Hyperbola

Hyperbola. This curve is formed by the intersection of a plane and a cone, the plane being parallel to the axis of the cone, Fig. 86d.

Like the parabola, the curve is not closed, the branches constantly diverging.

An hyperbola is defined as *a plane curve such that the difference between the distances from any point in the curve to two fixed points is equal to a given distance.*

The two fixed points are the *foci* and the line passing through them is the *transverse axis*, Fig. 89.

Rectangular Hyperbola. The form of hyperbola most used in Mechanical Engineering is called the rectangular hyperbola be-

Fig. 90. Construction of Rectangular Hyperbola

cause it is drawn with reference to rectangular coördinates. This curve is constructed as follows: In Fig. 90, $O X$ and $O Y$ are the two coördinate axes drawn at right angles to each other. These lines are also called *asymptotes*. Assume A to be a known point on the curve. Draw $A C$ parallel to $O X$ and $A D$ perpendicular to $O X$. Mark off any convenient points on $A C$ such as E, F, G, and H, and through these points draw EE', FF', GG', and HH', perpendicular to $O X$. Connect E, F, G, H, and C with O. Through the points of intersection of the oblique lines and the vertical line $A D$ draw the horizontal lines LL', MM', NN', PP', and QQ'. The first point on the curve is the assumed point A, the second point is R, the intersection of LL' and EE'. The third is the intersection S of MM' and FF'. The other points are found in the same way.

In this curve the products of the coördinates of all points are equal. Thus $LR \times RE' = MS \times SF' = NT \times TG'$.

ODONTOIDAL CURVES

Gears have their teeth cut by a machine so as to conform to certain shapes which will bring about smoothness of running when the gears are in mesh. The curves generally employed in shaping the gear teeth are the cycloidal and the involute.

Cycloidal Curves. *Cycloid.* The cycloid is a curve generated

by a point on the circumference of a circle which rolls on a straight line tangent to the circle, Fig. 91.

The rolling circle is called the *describing* or *generating circle,* the point on the circle, the *describing* or *generating point,* and the
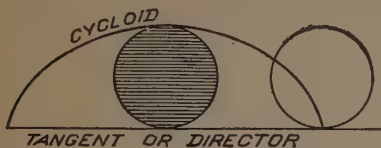
Fig. 91. Geometrical Construction
for a Cycloid
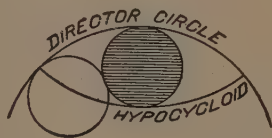
Fig. 92. Geometrical Construction
for a Hypocycloid

tangent along which the circle rolls, the *director.* In order that the curve described by the point may be a true cycloid the circle must roll without any slipping.

Hypocycloid. In case the generating circle rolls upon the inside of an arc or circle, the curve thus generated is a hypocycloid, Fig. 92   If the generating circle has a diameter equal to the radius of the director circle the hypocycloid becomes a straight line.

Epicycloid. If the generating circle rolls upon the *outside* of an arc or circle, called the *director circle,* the curve thus generated is an epicycloid, Fig. 93.

Involute Curves   If a thread or fine wire is wound around a cylinder or circle and then unwound, the end will describe an involute
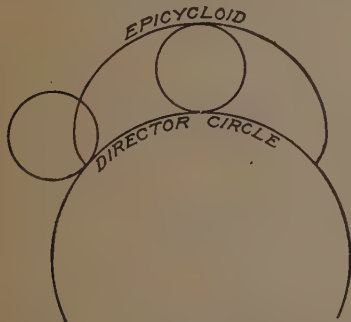
Fig. 93. Geometrical Construction
for an Epicycloid
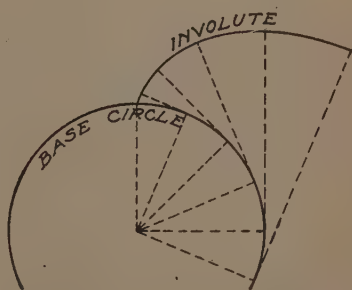
Fig. 94. Geometrical Construction
for an Involute

curve. The involute may be defined as *a curve generated by a point in a tangent rolling on a circle, known as the base circle,* Fig. 94.

The details of the construction of the ellipse, parabola, hyperbola, cycloid, and involute will be taken up in connection with the plates.

GEOMETRICAL PROBLEMS

The problems given in Plates IV to VIII inclusive have been chosen because of their particular bearing on the work of the mechanical draftsman. They should be solved with great care, as the principles involved will be used in later work.

PLATE IV

Penciling. The horizontal and vertical center lines and the border lines should be laid out in the same manner as in *Plate I*. Now measure off $2\frac{1}{4}$ inches on both sides of the vertical center line and through these points draw vertical lines as shown by the dot and dash lines, *Plate IV*. In locating the figures, place them a little above the center so that there will be room for the number of the problem.

Draw in lightly the lines of each figure with pencil and after the entire plate is completed, ink them. In penciling, all intersections must be formed with great care as the accuracy of the results depends upon it. Keep the pencil points in good order at all times and draw lines *exactly* through intersections.

Problem 1. *To bisect a given straight line.*

Draw the horizontal straight line $A\,C$ about 3 inches long. With the extremity A as a center and any convenient radius— about 2 inches—describe arcs above and below the line $A\,C$. With the other extremity C as a center and with the same radius draw similar arcs intersecting the first arcs at D and E. The radius of these arcs must be greater than one-half the length of the line in order that they may intersect. Now draw the straight line $D\,E$ passing through the intersections D and E. This line will cut $A\,C$ at its middle point F.

Therefore

$$A\,F = F\,C$$

Proof. Since the points D and E are equally distant from A and C a straight line drawn through them is perpendicular to $A\,C$ at its middle point F.

Problem 2. *To construct an angle equal to a given angle.*

Draw the line $O\,C$ about 2 inches long and the line $O\,A$ of about the same length. The angle formed by these lines may be

any convenient size—about 45 degrees is suitable. This angle
$A\ O\ C$ is the given angle.

Now draw $F\ G$, a horizontal line about $2\frac{1}{4}$ inches long, and let
F, the left-hand extremity, be the vertex of the angle to be con-
structed.

With O as a center and any convenient radius—about $1\frac{1}{2}$ inches
—describe the arc $L\ M$ cutting both $O\ A$ and $O\ C$. With F as a
center and the same radius draw the indefinite arc $O\ Q$. Now set
the compass so that the distance between the pencil and the needle
point is equal to the chord $L\ M$. With Q as a center and a radius
equal to $L\ M$ draw an arc cutting the arc $O\ Q$ at P. Through
F and P draw the straight line $F\ E$. The angle $E\ F\ G$ is the re-
quired angle since it is equal to $A\ O\ C$.

Proof. Since the chords of the arcs $L\ M$ and $P\ Q$ are equal,
the arcs are equal. The angles are equal because with equal radii
equal arcs are intercepted by equal angles.

Problems 3 and 4. *To draw through a given point a line parallel
to a given line.*

First Method. Draw the straight line $A\ C$ about $3\frac{1}{2}$ inches
long and assume the point P about $1\frac{1}{2}$ inches above $A\ C$. Through
the point P draw an oblique line $F\ E$ forming any convenient angle
—about 60 degrees—with $A\ C$. Now construct an angle equal to
$P\ F\ C$ having its vertex at P and the line $E\ P$ as one side. (See
Problem 2.) The straight line $P\ O$ forming the other side of the
angle $E\ P\ O$ will be parallel to $A\ C$.

Proof. If two straight lines are cut by a third making the
corresponding angles equal, the lines are parallel.

Second Method. Draw the straight line $A\ C$ about $3\frac{1}{4}$ inches
long and assume the point P about $1\frac{1}{2}$ inches above $A\ C$. With
P as a center and any convenient radius—about $2\frac{1}{2}$ inches—draw
the indefinite arc $E\ D$ cutting the line $A\ C$. Now with the same
radius and with D as a center, draw an arc $P\ Q$. Set the com-
pass so that the distance between the needle point and the pencil
is equal to the chord $P\ Q$. With D as a center and a radius equal
to $P\ Q$, describe an arc cutting the arc $E\ D$ at H. A line drawn
through P and H will be parallel to $A\ C$.

Proof. Draw the line $Q\ H$. Since the arcs $P\ Q$ and $H\ D$
are equal and have the same radii, the angles $P\ H\ Q$ and $H\ Q\ D$

are equal. Two lines are parallel if the alternate interior angles are equal.

Problems 5 and 6. *To draw a perpendicular to a line from a point in the line.*

First Method. WHEN THE POINT IS NEAR THE MIDDLE OF THE LINE.

Draw the line $A\,C$ about $3\frac{1}{2}$ inches long and assume the point P near the middle of the line. With P as a center and any convenient radius—about $1\frac{1}{4}$ inches—draw two arcs cutting the line $A\,C$ at E and F. Now with E and F as centers and any convenient radius —about $2\frac{1}{2}$ inches—describe arcs intersecting at O. The line $O\,P$ will be perpendicular to $A\,C$ at P.

Proof. The points P and O are both equally distant from E and F. Hence a line drawn through them is perpendicular to $E\,F$ at P.

Second Method. WHEN THE POINT IS NEAR THE END OF THE LINE.

Draw the line $A\,C$ about $3\frac{1}{2}$ inches long. Assume the given point P to be about $\frac{3}{4}$ inch from the end A. With any point D as a center and a radius equal to $D\,P$, describe an arc cutting $A\,C$ at E. Through E and D draw the diameter $E\,O$. A line from O to P is perpendicular to $A\,C$ at P.

Proof. The angle $O\,P\,E$ is inscribed in a semicircle; hence it is a right angle, and the sides $O\,P$ and $P\,E$ are perpendicular to each other.

Lettering. After completing these figures draw pencil lines for the lettering. Place the words *"Plate IV"* and the date and the name in the border, as in preceding plates. To letter the words "Problem 1," "Problem 2," etc., draw three horizontal lines $\frac{1}{4}$ inch, $\frac{3}{8}$ inch, and $\frac{7}{16}$ inch respectively, above the horizontal center line and the lower border line to serve as a guide for the size of the letters.

Inking. In inking *Plate IV*, ink in the figures first. Make the line $A\,C$, Problem 1, a full line as it is the given line; make the arcs and the line $D\,E$ dotted as they are construction lines. Similarly in Problem 2, make the sides of the angles full lines and the chord $L\,M$ and the arcs dotted. Follow the same plan in inking the lines of Problems 3, 4, 5, and 6. In Problem 6, ink in only that part of the circumference which passes through the points O, P, and E.

After inking the figures, ink in the heavy border line, and the lettering.

PLATE V

Penciling. In laying out the border lines and center lines follow the directions given for *Plate IV*. Draw the dot and dash lines in the same manner, as there are to be six problems on this plate.

Problem 7. *To draw a perpendicular to a line from a point without the line.*

Draw the straight line $A\,C$ about $3\frac{1}{4}$ inches long, and assume the point P about $1\frac{1}{2}$ inches above the line. With P as a center and any convenient radius—about 2 inches—describe an arc cutting $A\,C$ at E and F. The radius of this arc must always be such that it will cut $A\,C$ in two points; the nearer the points E and F are to A and C, the greater will be the accuracy of the work.

Now with E and F as centers and any convenient radius—about $2\frac{1}{4}$ inches—draw the arcs intersecting below $A\,C$ at T. A line through the points P and T will be perpendicular to $A\,C$. In case there is not room below $A\,C$ to draw the arcs, they may be drawn intersecting above the line as shown at N. Whenever convenient draw the arcs below $A\,C$ for greater accuracy.

Proof. Since P and T are both equally distant from E and F, the line $P\,T$ is perpendicular to $A\,C$.

Problems 8 and 9. *To bisect a given angle.*

First Method. WHEN THE SIDES INTERSECT.

Draw the lines $O\,C$ and $O\,A$—about 3 inches long—forming any angle of 45 to 60 degrees. With O as a center and any convenient radius—about 2 inches—draw an arc intersecting the sides of the angle at E and F. With E and F as centers and a radius of $1\frac{1}{2}$ or $1\frac{3}{4}$ inches, describe short arcs intersecting at I. A line $O\,D$, drawn through the points O and I, bisects the angle.

In solving this problem the arc $E\,F$ should not be too near the vertex if accuracy is desired.

Proof. The central angles $A\,O\,D$ and $D\,O\,C$ are equal because the arc $E\,F$ is bisected by the line $O\,D$. The point I is equally distant from E and F.

Second Method. WHEN THE LINES DO NOT INTERSECT.

Draw the lines $A\,C$ and $E\,F$ about 4 inches long making an

angle approximately as shown.   Draw A' C' and E' F' parallel to
A C and E F and at such equal distances from them that they will
intersect at O.   Now bisect the angle C' O F' by the method given in
Problem 8.   The line O R bisects the given angle.

Proof.   Since A' C' is parallel to A C and E' F' is parallel to
E F, the angle C' O F' is equal to the angle formed by the lines
A C and E F.   Hence as O R bisects angle C' O F' it also bisects
the angle formed by the lines A C and E F.

Problem 10.   *To divide a line into any number of equal parts.*

Let A C—about $3\frac{3}{4}$ inches long—be a given line.   Suppose
it is desired to divide it into 7 equal parts.   First draw the line A J
at least 4 inches long, forming any convenient angle with A C.   On
A J lay off, by means of the dividers or scale, points D, E, F, G, etc.,
each $\frac{1}{2}$ inch apart.   (If dividers are used, the spaces need not be
exactly $\frac{1}{2}$ inch.)   Draw the line J C and through the points D, E,
F, G, etc., draw lines parallel to J C.   These parallels will divide
the line A C into 7 equal parts.

Proof.   If a series of parallel lines, cutting two straight lines,
intercept equal distances on one of these lines, they also intercept
equal distances on the other.

Problem 11.   *To construct a triangle having given the three sides.*

Draw the three sides, A C, $2\frac{3}{4}$ inches long; E F, $1\frac{15}{16}$ inches long;
and M N, $2\frac{3}{16}$ inches long.

Draw R S equal in length to A C.   With R as a center and a
radius equal to E F describe an arc.   With S as a center and a radius
equal to M N draw an arc cutting the arc previously drawn, at T.
Connect T with R and S to form the triangle.

Problem 12.   *To construct a triangle having given one side and
the two adjacent angles.*

Draw the line M N $3\frac{1}{4}$ inches long and draw two angles A O D
and E F G about 30 degrees and 60 degrees respectively.

Draw R S equal in length to M N and with R as a vertex and
R S as one side construct an angle equal to A O D.   In a similar
manner construct at S an angle equal to E F G.   Draw lines from
R and S through the two established points until they meet at T.
The triangle R T S will be the required triangle.

Lettering.   Draw the pencil lines and put in the lettering as
in plates already drawn.

Inking. In inking *Plate V*, follow the principles previously used and do not make certain lines dotted until sure that they should be dotted.

After inking the figures, ink in the border lines and the lettering as already explained.

PLATE VI

Penciling. Lay out this plate in the same manner as the preceding plates.

Problem 13. *To describe an arc or circumference through three given points not in the same straight line.*

Locate the three points A, B, and C with a distance between A and B of about 2 inches and a distance between A and C of about $2\frac{1}{4}$ inches. Connect A and B and A and C. Erect perpendiculars to the middle points of $A B$ and $A C$ as explained in Problem 1. Now draw light pencil lines connecting the intersections I and J and E and F. These lines will intersect at O.

With O as a center and a radius equal to the distance $O A$, describe the circumference passing through A, B, and C.

Proof. The point O is equally distant from A, B, and C, since it lies in the perpendiculars to the middle points of $A B$ and $A C$. Hence the circumference will pass through A, B, and C.

Problem 14. *To inscribe a circle in a given triangle.*

Draw the triangle $L M N$ of any convenient size. $M N$ may be made $3\frac{1}{4}$ inches, $L M$, $2\frac{3}{4}$ inches, and $L N$, $3\frac{1}{2}$ inches. Bisect the angles $M L N$ and $L M N$ by the method used in Problem 8. The bisectors $M I$ and $L J$ intersect at O, which is the center of the inscribed circle. The radius of the circle is equal to the perpendicular distance from O to one of the sides.

Proof. The point of intersection of the bisectors of the angles of a triangle is equally distant from the sides.

Problem 15. *To inscribe a regular pentagon in a given circle.*

With O as a center and a radius of about $1\frac{1}{2}$ inches, describe the given circle. With the T-square and triangles draw the center lines $A C$ and $E F$ perpendicular to each other and passing through O. Bisect one of the *radii*, $O C$, at H and with this point as a center and a radius $H E$, describe the arc $E P$. This arc cuts the diameter $A C$ at P. With E as a center and a radius $E P$, draw arcs cutting

the circumference at L and Q. With the same radius and centers at L and Q, draw the arcs cutting the circumference at M and N.

The pentagon is completed by drawing the chords $E L$, $L M$, $M N$, $N Q$, and $Q E$.

Problem 16. *To inscribe a regular hexagon in a given circle.*

With O as a center and a radius of $1\frac{3}{8}$ inches draw the given circle. With the T-square draw the diameter $A D$. With D as a center, and a radius equal to $O D$, describe arcs cutting the circumference at C and E. Now with C and E as centers and the same radius, draw the arcs, cutting the circumference at B and F. Draw the hexagon by joining the points thus formed.

Therefore, in order to inscribe a regular hexagon in a circle, mark off chords equal in length to the radius.

To inscribe an equilateral triangle in a circle the same method may be used, the triangle being formed by joining the opposite vertices of the hexagon.

Proof. Since the triangle $O C D$ is an equilateral triangle by construction, the angle $C O D$ is one-third of two right angles and one-sixth of four right angles. Hence arc $C D$ is one-sixth of the circumference and the chord is a side of a regular hexagon.

Problem 17. *To draw a line tangent to a circle at a given point on the circumference.*

With O as a center and a radius of about $1\frac{1}{4}$ inches draw the given circle. Assume some point P on the circumference and join the point P with the center O. By the method given in Problem 6, *Plate IV*, construct a perpendicular to $P O$, which perpendicular will be the desired tangent to the circle at the point P.

Proof. A line perpendicular to a radius at its extremity is tangent to the circle.

Problem 18. *To draw a line tangent to a circle from a point outside the circle.*

With O as a center and a radius of about 1 inch draw the given circle. Assume P some point outside of the circle about $2\frac{1}{2}$ inches from the center. Draw a straight line passing through P and O. Bisect $P O$ and with the middle point F as a center describe the circle passing through P and O. Draw a line from P through the intersection of the two circumferences C. The line $P C$ is tangent to the given circle. Similarly $P E$ is tangent to the circle.

Proof. The angle $P\,C\,O$ is inscribed in a semicircle and hence is a right angle. Since $P\,C\,O$ is a right angle, $P\,C$ is perpendicular to $C\,O$. The perpendicular to a radius at its extremity is tangent to the circumference.

Inking. In inking *Plate VI*, the same method should be followed as in previous plates.

PLATE VII

Penciling. Lay out this plate in the same manner as the preceding plates.

Problems 19 and 20. *To draw an ellipse when the axes are given.*

First Method. Draw the lines $L\,M$ and $C\,D$ about $3\frac{1}{4}$ and $2\frac{1}{4}$ inches long respectively, making $C\,D$ perpendicular to $L\,M$ at its middle point P and having $C\,P = P\,D$. The two lines, $L\,M$ and $C\,D$, are the axes. With C as a center and a radius $L\,P$ equal to one-half the major axis, draw the arc, cutting the major axis at E and F. These two points are the foci.

Now locate several points on $P\,M$, such as A, B, and G. With E as a center and a radius equal to $L\,A$, draw arcs above and below $L\,M$. With F as a center and a radius equal to $A\,M$ describe short arcs cutting those already drawn as shown at N. With E as a center and a radius equal to $L\,B$ draw arcs above and below $L\,M$ as before. With F as a center and a radius equal to $B\,M$, draw arcs intersecting those already drawn as shown at O. The point R and others are found by repeating the process. The student is advised to find at least 12 points on the curve—6 above and 6 below $L\,M$. These 12 points with L, C, M, and D will enable him to draw the curve.

After locating these points, draw a free-hand curve passing through them.

Second Method. Draw the two axes $A\,B$ and $P\,Q$ in the same manner as in the first method. With O as a center and a radius equal to one-half the major axis, describe a circle. Similarly with the same center and a radius equal to one-half the minor axis, describe another circle. Draw any radii such as $O\,C$, $O\,D$, $O\,E$, $O\,F$, etc., cutting both circumferences. These radii may be drawn with the 60 and 45 degree triangles. From C, D, E, and F, the points of intersection of the radii with the large circle, draw *vertical* lines and from C', D', E', and F', the points of intersection of the radii with

the small circle, draw *horizontal* lines   The intersections of these lines are points on the ellipse.

Draw a free-hand curve passing through these points; about five points in each quadrant will be sufficient.

Problem 21.   *To draw an ellipse by means of a trammel.*

As in Problems 19 and 20, draw the major and minor axes, *U V* and *X Y*. Take a slip of paper having a straight edge and mark off *C B* equal to one-half the major axis, and *D B* equal to one-half the minor axis.   Place the slip of paper in various positions keeping the point *D* on the major axis and the point *C* on the minor axis.   If this is done, the point *B* will mark various points on the curve.   Find as many points as necessary and sketch the ellipse.

Problem 22.   *To draw a spiral of one turn in a circle.*

Draw a circle with the center at *O* and a radius of $1\frac{1}{2}$ inches. Locate twelve points, $\frac{1}{8}$ inch apart on the radius *O A* and draw circles through these points.   Now, by means of the 30-degree triangle, draw radii *O B*, *O C*, *O D*, etc., 30 degrees apart, thus dividing the circle into 12 equal parts.

The points on the spiral are now located; the first is at the center *O;* the next is at the intersection of the line *O B* and the first circle; the third is at the intersection of *O C* and the second circle; the other points are located in the same way.   Sketch in pencil a smooth curve passing through these points.

Problem 23.   *To draw a parabola when the abscissa and ordinate are given.*

Draw the straight line *A B*—about three inches long—as the axis, or *abscissa* of the parabola.   At *A* and *B* draw the lines *C D* and *E F* perpendicular to *A B*, and with the T-square draw *E C* and *F D*, $1\frac{1}{2}$ inches above and below *A B*, respectively.   Let *A* be the vertex of the parabola.   Divide *A E* and *E C* into the same number of equal parts.   Through *R*, *S*, *T*, *U*, and *V*, draw horizontal lines and connect *L*, *M*, *N*, *O*, and *P*, with *A*.   The intersections of the horizontal lines with the oblique lines are points on the curve. For instance, the intersection of *A L* and the line *V* is one point and the intersection of *A M* and the line *U* is another.

The lower part of the curve *A D* is drawn in a similar manner.

Problem 24.   *To draw a hyperbola when the abscissa E X, the ordinate A E, and the diameter X Y are given.*

Draw $E\,F$ about 3 inches long and mark the point X, 1 inch from E and the point Y, 1 inch from X. With the triangle and T-square, draw the rectangles $A\,B\,D\,C$ and $O\,P\,Q\,R$ such that $A\,B$ is 1 inch in length and $A\,C$, 3 inches in length. Divide $A\,E$ and $A\,B$ into the same number of equal parts. Connect Y with the points T, U, and V, on $A\,F$, and connect X with L, M, and N, on $A\,B$. The first point on the curve is at A; the next is at the intersection of $T\,Y$ and $L\,X$; the third is at the intersection of $U\,Y$ and $M\,X$. The remaining points are found in the same manner. Repeat the process for $X\,C$ and the right-hand curve $P\,Y\,Q$.

Inking. In inking the figures on this plate, use the French or irregular curve and make full lines for the curves and their axes. Dot the construction lines as usual. Ink in all the construction lines used in finding one-half of a curve, and in Problems 19, 20, 23, and 24 leave all construction lines in *pencil* except those inked. In Problems 21 and 22 erase all construction lines not inked. The trammel used in Problem 21 may be drawn in the position shown, or outside of the ellipse in any convenient place.

The same lettering should be done on this plate as on previous plates

PLATE VIII

Penciling. In laying out *Plate VIII*, draw the border lines and horizontal and vertical center lines as in previous plates, dividing the plate into four spaces.

Problem 25. *To construct a cycloid when the diameter of the generating circle is given.*

With O' as a center and a radius of $\frac{7}{8}$ inch draw a circle, and, tangent to it, draw the indefinite horizontal straight line $A\,B$. Divide the circle into any number of equal parts—12 for instance—and through these points of division C, D, E, F, etc., draw horizontal lines. Now with the dividers set so that the distance between the points is equal to the chord of the arc $C\,D$, mark off the points L, M, N, O, P, on the line $A\,B$, commencing at the point H. At these points erect perpendiculars to the center line $G\,O'$ which is the line of centers of the generating circle as it rolls along the line $A\,B$. With the intersections Q, R, S, T, etc., as centers describe arcs of circles as shown. The points on the cycloid will be the intersections of

these arcs and the horizontal lines drawn through the points C, D, E, F, etc. Thus the intersection of the arc whose center is Q and the horizontal line through C is a point I on the curve. Similarly, the intersection of the arc whose center is R and the horizontal line through D is the point J on the curve. The remaining points on the left, as well as those on the right, are found in the same manner. To obtain great accuracy in this curve, the circle should be divided into a large number of equal parts, because the greater the number of divisions the less the error due to the difference in length between a chord and its arc.

Problem 26. *To construct an epicycloid when the diameter of the generating circle and the diameter of the director circle are given.*

The epicycloid and the hypocycloid may be drawn in the same manner as the cycloid if arcs of circles are used in place of the horizontal lines. With O as a center and a radius of $\frac{3}{4}$ inch describe a circle. Draw the diameter $E\,F$ of this circle and produce $E\,F$ to G such that the line $F\,G$ is $2\frac{3}{4}$ inches long. With G as a center and a radius $F\,G$, describe the arc $A\,B$ of the director circle. With the same center G, draw the arc $P\,Q$ which will be the path of the center of the generating circle as it rolls along the arc $A\,B$. Now divide the generating circle into any number of equal parts—twelve for instance—and through the points of division H, I, L, M, and N, draw arcs having G as a center. With the dividers set so that the distance between the points is equal to the chord $H\,I$, mark off distances on the director circle $A\,F\,B$. Through these points of division R, S, T, U, etc., draw radii intersecting the arc $P\,Q$ in the points R', S', T', etc., and with these points as centers describe arcs of circles as in Problem 25. The intersections of these arcs with the arcs already drawn through the points H, I, L, M, etc., are points on the epicycloid. Thus the intersection of the circle whose center is R' with the arc drawn through the point H is a point upon the curve. Also the arc whose center is S' with the arc drawn through the point I is another point on the curve. The remaining points are found by repeating this process.

Problem 27. *To draw an hypocycloid when the diameter of the generating circle and the radius of the director circle are given.*

With O as a center and a radius of 4 inches describe the arc $E\,F$, which is the arc of the director circle. Now with the same

center and a radius of $3\frac{1}{4}$ inches, describe the arc A B, which is the line of centers of the generating circle as it rolls on the director circle. With O' as a center and a radius of $\frac{3}{4}$ inch describe the generating circle. As before, divide the generating circle into any number of equal parts—12, for instance—and with these points of division L, M, N, O, etc., draw arcs having C as a center. Upon the arc E F, lay off distances Q R, R S, S T, etc., equal to the chord Q L. Draw radii from the points R, S, T, etc., to the center of the director circle O and describe arcs of circles having a radius equal to the radius of the generating circle, using the points G, I, J, etc., as centers. As in Problem 26, the intersections of the arcs are the points on the hypocycloid. By repeating this process, the right-hand portion of the curve may be drawn.

Problem 28. *To draw the involute of a circle when the diameter of the base circle is known.*

With the point O as a center and a radius of 1 inch, describe the base circle. Divide the circle into any number of equal parts—16, for instance—and draw radii to the points of division. At the point D, draw a light pencil line perpendicular to O D. This line will be tangent to the circle. Similarly at the points E, F, G, H, etc., draw tangents to the circle. Set the dividers so that the distance between the points will be equal to the chord of the arc C D, and measure this distance from D along the tangent. From the point E, measure on the tangent a distance equal to two of these chords; from the point F, three divisions; and from the point G, four divisions. Similarly, measure distances on the remaining tangents, each time adding the length of the chord. This will give the points L, M, N, P, etc., to T. The curve drawn through these points will be the involute of the circle.

Inking. Observe the same rules in inking *Plate VIII* as were given for *Plate VII*. In Problems 25 and 26 the arcs and lines used in locating the points of the other half of the curve may be left in pencil. In Problem 28, all construction lines should be inked. After completing the problems the same lettering should be done on this plate as on previous plates.

PLATE IV
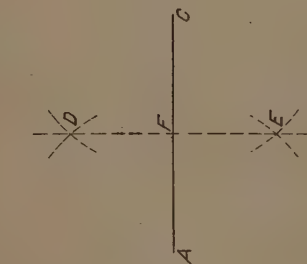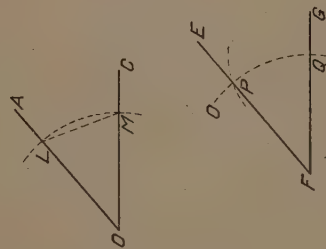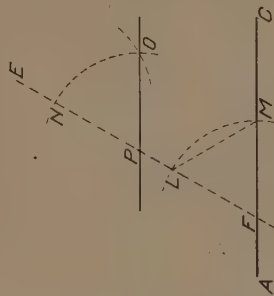
Problem 1

Problem 2

Problem 3
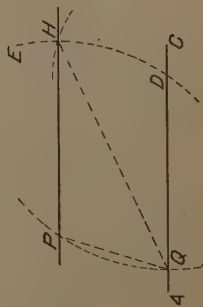
Problem 4

Problem 5

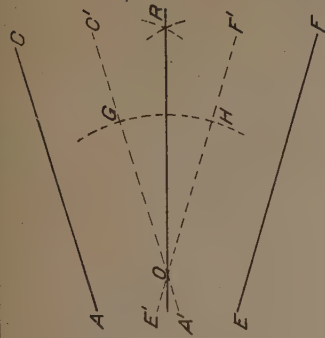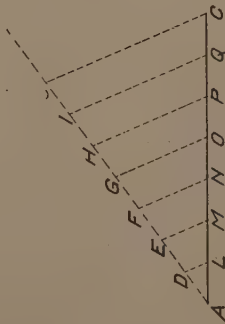Problem 6
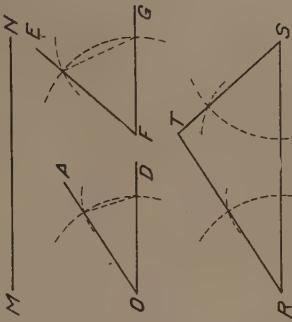
JANUARY 14, 1910.   HERBERT CHANDLER, CHICAGO, ILL.

PLATE V
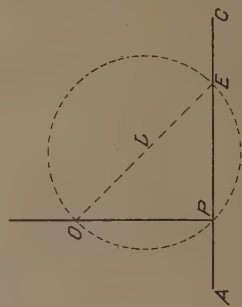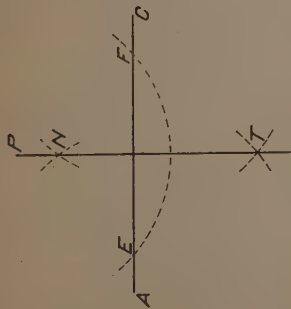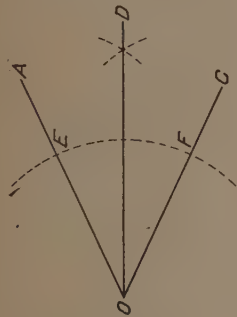
Problem 9

Problem 8

Problem 7

Problem 12

Problem 11

Problem 10

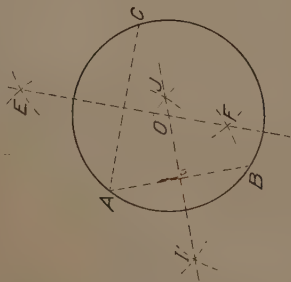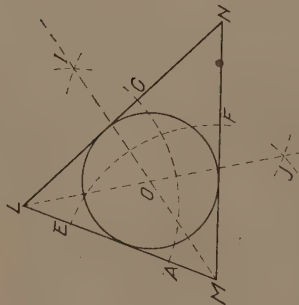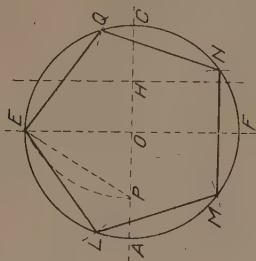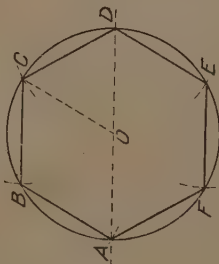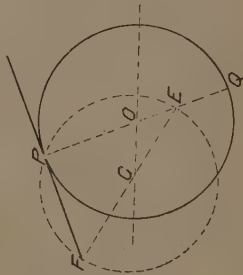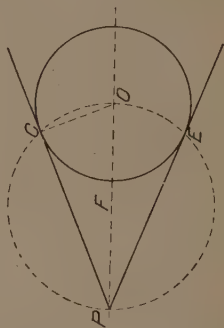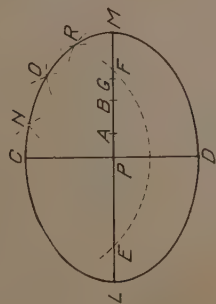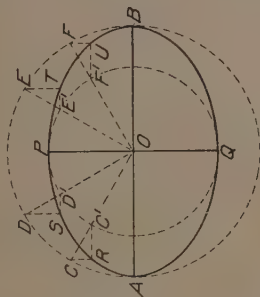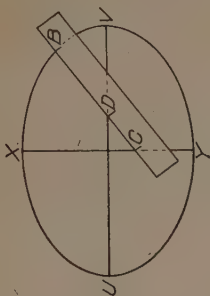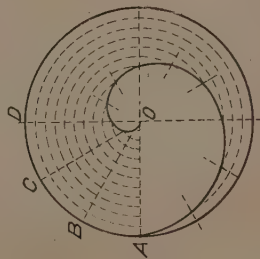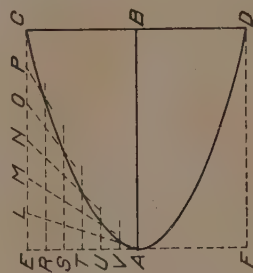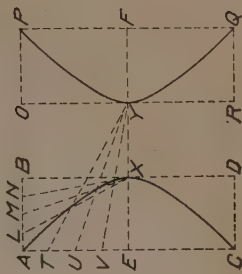JANUARY 19, 1910. HERBERT CHANDLER, CHICAGO, ILL.

PLATE VI

Problem 15

Problem 18

Problem 14

Problem 17

Problem 13

Problem 16
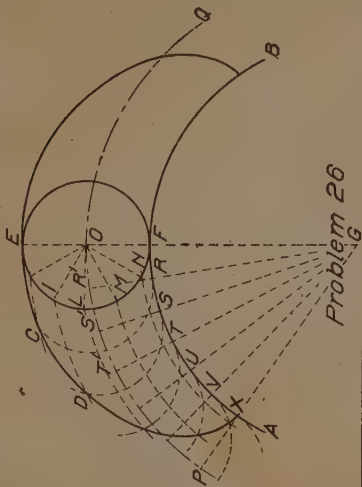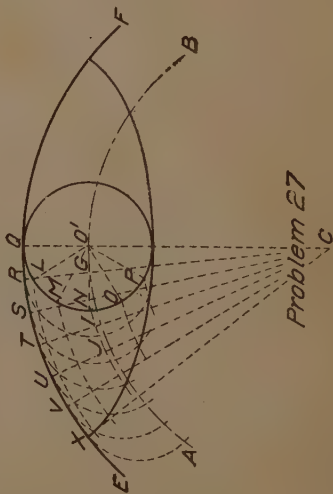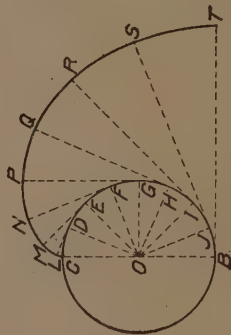
JANUARY 29, 1910.   HERBERT CHANDLER, CHICAGO, ILL.

PLATE VII

Problem 19

Problem 20

Problem 21

Problem 22

Problem 23

Problem 24

FEBRUARY 4, 1910.   HERBERT CHANDLER, CHICAGO, ILL.

PLATE VIII

Problem 26

Problem 28

Problem 25

Problem 27